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Introduction to Chapters VII and IX of Augustin Cournot, *Researches into the Mathematical Principles of the Theory of Wealth*

Michael A. Salinger

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I. Introduction

In November 2007, the European Commission accepted a set of guidelines concerning its review of non-horizontal mergers. The section on conglomerate mergers contains a discussion of the possibility that merging firms will bundle their products together. It reads, in part:

> "[W]hen producers of complementary goods are pricing independently, they will not take into account the positive effect of a drop in the price of their product on the sales of the other product. Depending on the market conditions, a merged firm may internalise this effect and may have a certain incentive to lower margins if this leads to higher overall profits (this incentive is often referred to as the "Cournot effect")."[1]

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[1] Commission Notice, Guidelines on the assessment of non-horizontal mergers under the Council Regulation on the control of concentrations between undertakings ¶117 (Nov. 28, 2007), *at* http://ec.europa.eu/comm/competition/mergers/legislation/nonhorizontalguidelines.pdf (last visited Apr. 8, 2008).

❘ The author is Professor of Economics at Boston University School of Management.

The Cournot to which this passage refers is Augustin Cournot, the nineteenth century French mathematician whose treatise *Recherches sur les principes mathématiques de la théorie des richesses* (*Researches into the Mathematical Principles of the Theory of Wealth*) was published in 1838.[2] The two chapters of the English translation of the book reprinted in this issue are the two that are most relevant for industrial economics and antitrust enforcement. The first of these chapters presents what is known as the Cournot oligopoly model. The second concerns pricing decisions by monopolist sellers of complementary products and is the basis for the "Cournot" effect referenced in the EC's non-horizontal merger guidelines.

While these chapters cover topics in what is now known as industrial economics, the book as a whole is not a precursor of modern industrial economics texts. Economists today would characterize the subject of the book as price theory (at the University of Chicago and like-minded places) or microeconomic theory (everywhere else). The reference to wealth is a bit misleading, as it seems to suggest a treatment of saving and investment. For Cournot, what made something a source of wealth was the ability to exchange it in a market. Therefore, an individual's wealth depends critically on the price he can receive for whatever he has to sell, hence the link between the reference to wealth in the title and the book's focus on prices.

Even if the reference to wealth in the title misleads modern readers, the reference to mathematics will not. The book is not highly sophisticated by modern standards in economics, but the treatment is most definitely mathematical. Readers comfortable with mathematics (i.e., most antitrust economists and some antitrust attorneys) will find the chapters republished in this issue to be a real pleasure. The chapters will be more of a challenge to those who are not "fluent" in mathematics (i.e., most, but not all, antitrust attorneys); but, as I try to explain in Section III of this paper, they are worth reading while skipping over the equations. Before turning to that explanation, I briefly address what, as an economist, I found most interesting.

II. Interest to Economists

My first reaction upon revisiting this book after many years is how far ahead of his time Cournot was. Few modern economists question the proposition that mathematics is an essential tool of economic theory. Indeed, to be a modern "economic theorist", one must, virtually by definition, construct mathematical models. The likes of John Kenneth Galbraith and Joseph Schumpeter no longer count as economic theorists. This was not the case when Cournot wrote. The

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2   AUGUSTIN COURNOT, RESEARCHES INTO THE MATHEMATICAL PRINCIPLES OF THE THEORY OF WEALTH (N.T. Bacon trans., Augustus M. Kelley, 1971) (1838) [hereinafter Cournot (1838)]. Chapters VII and IX are reprinted in this issue, 4(1) COMPETITION POL'Y INT'L 283-305 (Spring 2008) [hereinafter Cournot reprint].

preface to Cournot's book takes on what he views as a condemnation of the use of mathematics by the "theorists" of political economy of his time such as Adam Smith, Jean-Baptiste Say, and David Ricardo. In Cournot's view, Smith and Say "preserved all the beauties of a purely literary style;"[3] whereas Ricardo, who did rely on algebra, "disguised it under arithmetical calculations of tiresome length."[4] The notion that mathematics was a useful tool for exploring the generality of economic propositions was not even the cutting edge of economics until the late nineteenth century. For economists, this book, published in 1838, reads remarkably well today.

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Given how innovative Cournot's approach was, a second remarkable feature is how much progress he was able to make. Cournot was the first economist to write a mathematical equation for the demand curve that linked the quantity demanded to the price charged.[5] From such a rudimentary beginning, it would have been impressive had he merely derived the monopoly price. His contributions far exceeded that, however. Not only did he derive a coherent model of duopoly, he generalized the model to an arbitrary number of firms,[6] allowed for the possibility that the firms would have different costs, and showed that perfect competition is the limit of the Cournot oligopoly model as the number of firms goes to infinity.[7] This is just what he covered in two chapters, neither of which includes the chapter that arguably contains the book's most enduring insight.

Finally, Cournot's analysis was the first example of mathematical game theory (i.e., the mathematical analysis of interdependent decisions). In both the chapters reprinted in this issue, the equilibria are examples of what are now referred to as Nash equilibria. Cournot does not seem to have appreciated the inherent indeterminacy of oligopolistic outcomes. In a paragraph to which I return later in this paper, he clearly understood the incentive for oligopolists to coordinate on outputs and prices, but he does not seem to have anticipated the possibility that they could do so without explicit coordination.

It is interesting that he chose to model duopolists as choosing outputs and complementary monopolists as choosing prices. Cournot does not explain the difference in modeling choices, but he does point out that his arguments gener-

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3   Cournot (1838), *id.* at 4.

4   *Id.*

5   *Id.* at ch. IV, "Of the Law of Demand."

6   Cournot reprint, *supra* note 2, at 287.

7   Cournot (1838), *supra* note 2, at ch. VIII, "Of Unlimited Competition."

ally require that the mathematical functions underlying the analysis be continuous.[8] With homogenous products, the profit functions for oligopolists choosing prices (taking the competitor's price as given) are discontinuous at the competitor's price. In a model of complementary monopolists choosing outputs, the payoff functions are continuous, but they have a kink at the quantity chosen by the other firm. This kink then complicates the determination of the division of the value between stages, one of the topics Cournot sought to address. The differences in the choices might well have been for purely mathematical reasons.

Regardless of his reasons, from the standpoint of economic theory, how to combine the two models presented here (i.e., how to construct a model with a small number of suppliers at successive or complementary stages when some of the firms produce at both), is quite a challenging problem. As part of the development of its guidelines, the European Commission commissioned Professor Jeffrey Church to survey the theoretical literature for possible insights to serve as a basis for the guidelines.[9] That report contains an extensive discussion of attempts to combine the two models. As a thorough review of the issues is beyond the scope of this introduction, interested readers should read the Church report. Suffice it to say, Cournot's modeling choices glossed over some deep issues.

This last point is no more a criticism than it would be a critique of the Wright brothers that their plane was primitive by modern standards. First efforts are supposed to be primitive. In Cournot's case, he advanced the ball so far that it took the profession many years to catch up and advance it further. Economists will marvel at these chapters, and I suspect many who have not done so already will be inspired to read the book in its entirety.

III. Interest to Antitrust Practitioners

Less mathematically inclined readers might not share economists' wonder and awe at Cournot's achievement. Indeed, some might suspect that Cournot was where all the trouble began. But, they too should marvel at the work. If Cournot's insights had been appreciated before they in fact were, many of the false steps made by early antitrust analysis might have been avoided.

The first of the chapters reprinted in this issue, chapter VII, "Of the Competition of Producers," contains analysis that is at the heart of much of antitrust enforcement. It follows the derivation in chapter V of the monopoly price and the analysis in chapter VI of the effect of tax on the price of a monopolized commodity. Chapter VII begins with an extension of the number of firms

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8 *Id.* at 49.

9 Jeffrey Church, The Impact of Vertical and Conglomerate Mergers on Competition, Report for DG Competition, European Commission (Sep. 2004), *available at* http://ec.europa.eu/comm/competition/mergers/studies_reports/merger_impact.pdf.

in the market from one to two. Starting with the end of the second paragraph, Cournot hones in on a key issue. He is analyzing the market outcome under duopoly assuming firms act independently. As he correctly points out, the firms would choose the monopoly price if they could come to an agreement. The following mathematical analysis culminates at the end of paragraph 45. Mathematically, the key is that the "root of equation (3) is always smaller than that of equation (4)."[10] The economic significance is that "the result of competition is to reduce prices."[11] Even those who do not understand how a mathematical equation could have this meaning will be hard-pressed to imagine a more concise statement of the philosophy underlying antitrust laws.

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Do not, however, skip to the bottom line, as some of the intermediate argument contains great insight. In the two full proceeding paragraphs, Cournot considers the question of why producers do not choose the price that maximizes industry profits (i.e., "the value of p derived from equation (4)"). He goes on to explain that if one firm were indeed to produce half the monopoly output, then:

"the other will be able to fix his own production at a higher or lower rate with a *temporary benefit*. To be sure, he will soon be punished for his mistake, because he will force the first producer to adopt a new scale of production which will react unfavourably on producer (2) himself. But these successive reactions ... will separate them further and further from it. In other words, this condition [...], although most favourable for both producers, [] can only be maintained by means of a formal engagement."[12]

Either because the possibility of tacit collusion did not occur to him or because he was skeptical of its practical importance, Cournot apparently believed competitors would have to meet to fix prices to overcome what we now refer to as "Prisoner's Dilemma".

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10 Cournot reprint, *supra* note 2, at 287.

11 *Id.*

12 *Id.* at 286. It is puzzling that Cournot did not realize that the competitor would necessarily choose a higher rate of output.

While the conclusion in chapter VII about the relationship between competition and prices was certainly a fundamental insight, the insight of chapter IX, "Of the Mutual Relations of Producers," may have been Cournot's most impressive contribution to antitrust. (If the depth of an insight is judged by how long it took for its importance to be recognized, chapter IX easily trumps chapter VII.) The problem covered by the chapter concerns the pricing of two or more goods that are combined (in fixed proportions) into a third good. The example he gives is the pricing of copper and zinc for the purposes of making brass. Having set up the problem, he goes through a mathematical analysis that in many ways parallels the derivation in chapter VII leading to equation (c), which he then compares to equation (c′).[13] Commenting on the comparison, Cournot notes that "there is this essential and very remarkable difference, that the root of (c) is always greater than that of the equation (c′) …" To translate, the "root of (c)" means the value of p, the price of the final good (brass) with the two monopolized inputs (zinc and copper), that causes equation (c) to be true (i.e., the price consumers would have to pay for brass if the zinc and copper monopolists set their prices independently). The comparison with the "root of (c′)" is with the price of the final good if they were to set their prices cooperatively. The "very remarkable difference" is:

"[T]he composite commodity will always be made more expensive, by reason of separation of interests than by reason of the fusion of monopolies. An association of monopolists, working for their own interest, in this instance will also work *for the interest of consumers*, which is exactly the opposite of what happens with competing producers."[14] (emphasis added)

This result forms the basis for Cournot's admonition in the chapter's introductory paragraph that "[T]he influence of the *mutual relations* of producers of different articles … must not be confounded with that of the *competition* of producers."[15]

The general principle Cournot derived is sometimes referred to as "double marginalization." Cournot showed that it was quite a general principle. But a principle that holds true in general must also hold true in a specific case, and specific cases can be easier to understand. Suppose that in a particular locality, one firm has a state-franchised, unregulated monopoly on peanut butter and another

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13 *Id.* at 295.

14 *Id.*

15 *Id.* at 292.

has a state-franchised, unregulated monopoly on jelly. Suppose further that residents of this locality are quite fussy about their peanut butter and jelly sandwich-

es, insisting that they contain exactly one ounce of peanut butter and one ounce of jelly. Production costs for peanut butter and jelly are both $2.00 per 8 ounce jar, and the monopolist sellers of both currently charge $4.00. They both sell 2 million jars per year, which yields a profit of $4 million each.

Consumer demand for peanut butter and jelly sandwiches depends on prices. Because consumers purchase them in fixed proportions to make the sandwiches, the demand for each good depends on both prices. Currently, the peanut butter and jelly inputs combined cost consumers $1 per sandwich.[16] Suppose that if the cost to consumers were to drop to $0.90 per sandwich, they would buy 2.6 million jars of both goods.

One way for the price to drop to $0.90 per sandwich would be for the jelly producer to lower its price to $3.20 per 8 ounce jar or $0.40 per ounce (i.e., per sandwich). If it did so, its profits would be ($3.20 – $2.00) × 2.6 million = $3.12 million, or $0.88 million less than it earns when it charges $4.00 per jar. Therefore, it has no incentive to cut the price. The same analysis applies to the peanut butter producer. Suppose, instead, that both monopolists cut their prices to $3.60 per jar, which would also reduce the cost to consumers to $0.90 per sandwich for the peanut butter and jelly input. Then, profits for both would be ($3.60 – $2.00) × 2.6 million = $4.16 million, which is more than the both earn when they both charge $4.00 per jar. Absent a merger or agreement on prices, both have an incentive to keep their price at $4.00 per jar even though a price reduction by both is in their mutual interest. If they were to merge, they would internalize the effect the price reduction of one good has on sale of the other and, therefore, would have an incentive to lower prices.

Today, the fundamental distinction between horizontal and vertical effects is an insight widely accepted by antitrust practitioners. Such was not always the case, however. In 1968, the U.S. Supreme Court ruled in Albrecht v. Herald Co.[17] that maximum resale price maintenance was to be treated as a per se violation of the antitrust laws as if it were equivalent to horizontal price-fixing by competitors. Had the Supreme Court understood Cournot's insight, it would not have taken until its 1997 State Oil Company v. Khan[18] decision to recognize the

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16 Given the assumption that a sandwich requires an ounce of peanut butter and an ounce of jelly, buying one eight-ounce jar of each (for a total of $8) provided the peanut butter and jelly needed for eight sandwiches.

17 Albrecht v. Herald Co., 390 U.S. 145 (1968).

18 State Oil Co. v. Khan, 522 U.S. 3 (1997).

difference between vertical and horizontal price-fixing. One can, of course, debate whether Cournot provided a complete justification for the more recent *Leegin* decision,[19] overturning the per se rule against minimum resale price maintenance. Cournot's model explains why a monopolist manufacturer might want to limit the price charged by its retailers, but not why it would set a price floor. Yet, the search for the subsequent explanations[20] arguably started with Cournot's insight that the "*mutual relations* of producers" are fundamentally different from the "*competition* of producers."

   It is sometimes said that the mark of a great piece of economic theory is that readers consider its central proposition to be obviously wrong before reading it and obviously right after reading it. Since thirty years passed before virtually anyone acknowledged Cournot's contribution and many more years before its importance was widely recognized, this hallmark of greatness did not occur for each individual reader. But the current widespread view that Cournot's insights are obvious even by people who are challenged by his mathematical approach is testimony to the enduring greatness of his achievement. ▼

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19 Leegin Creative Leather Products, Inc. v. PSKS, Inc., 551 U.S. ___ , 127 S. Ct. 2705 (2007).

20 *See* Lester G. Telser, *Why Should Manufacturers Want Fair Trade?*, 3 J.L. ECON. 86-105 (1960); and Howard P. Marvel & Stephen McCafferty, *Resale Price Maintenance and Quality Certification*, 15(3) RAND J. ECON. 346-59 (1984).