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Reconciling the Opposing Views of Critical Elasticity

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Reconciling the Opposing Views of Critical Elasticity

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I. INTRODUCTION

Market definition is the core of antitrust analysis, and the concepts of "critical elasticity" and "critical loss" have often been applied to the task of defining relevant antitrust markets, in both differentiated-product and homogenous-product scenarios. The critical elasticity is that elasticity of demand that is just high enough to prevent a hypothetical monopolist from profitably increasing price by a threshold "small but significant" amount; critical loss is the fraction of sales lost by the hypothetical monopolist, as implied by the critical elasticity. Evidence that the actual demand elasticity exceeds the critical elasticity indicates that the product in question is not a relevant market.

In recent articles, several researchers have offered an alternative approach to the issue of critical elasticity.[2] Their view is that the traditional application of the concept is flawed and that critical elasticity calculations contain information about actual elasticities of demand. We believe that alternative view to be half wrong. While it offers a misleading interpretation of critical elasticity analysis, the alternative view does reveal a flaw in the conventional approach—a flaw that proponents of the conventional approach have failed to recognize. Here

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[2] The list of relevant articles in this debate includes:

- Malcolm Coate & Jeffrey Fischer, *Critical Loss: Implementing the Hypothetical Monopolist Test*, GLOBAL COMPETITION POL'Y, (April 2008);
- Kevin Murphy & Robert Topel, *Critical Loss Analysis in the Whole Foods Case*, GLOBAL COMPETITION POL'Y, (March 2008);
- Gregory Werden, *Beyond Critical Loss: Properly Applying the Hypothetical Monopolist Test*, GLOBAL COMPETITION POL'Y, (February 2008);
- Joseph Farrell & Carl Shapiro, *Improving Critical Loss Analysis*, ANTITRUST SOURCE, (February 2008);
- Daniel O'Brien & Abraham Wickelgren, *The State of Critical Loss Analysis: Reply to Scheffman and Simons*, ANTITRUST SOURCE, (March 2004);
- Michael Katz & Carl Shapiro, *Further Thoughts on Critical Loss*, ANTITRUST SOURCE, (March 2004);
- David Scheffman & Joseph Simons, *The State of Critical Loss Analysis: Let's Make Sure We Understand the Whole Story*, ANTITRUST SOURCE, (November 2003);
- Daniel O'Brien & Abraham Wickelgren, *A Critical Analysis of Critical Loss Analysis*, ANTITRUST L.J., (2003); and
- Michael Katz & Carl Shapiro, *Critical Loss: Let's Tell the Whole Story*, ANTITRUST, (Spring 2003).

we present a revised and more general critical elasticity model, one that might reconcile the competing arguments. The new model also implies substantially higher critical elasticities than previous models would indicate.

II. CRITICAL ELASTICITY REVIEWED

The concepts of critical elasticity and critical loss have been used to determine the appropriate relevant market for antitrust purposes, in a way that is intended to be consistent with the approach taken by the U.S. federal antitrust authorities.[3] As noted, the critical elasticity is the elasticity of demand that is just high enough to prevent a hypothetical monopolist from profitably increasing price—by whatever percentage represents the threshold "small but significant non-transitory" amount. Evidence that the hypothetical monopolist faces a demand elasticity greater than the critical elasticity indicates that the product in question is not a relevant market. Critical loss is the fraction of sales lost by the hypothetical monopolist, as implied by the critical elasticity.

The basic result of critical elasticity analysis is straightforward. For any given price increase, the critical elasticity decreases as the initial price-cost margin increases. A monopolist more readily gives up low margin sales to achieve a higher price on remaining sales, whereas high margin sales are more costly to relinquish. Thus, when the initial margin is low, a higher elasticity is necessary to prevent a monopolist from raising price by a given percentage. In previous work, we derived the following critical elasticity formula: the critical elasticity equals

$$1 / (2t + m)$$

where m is the initial margin over short-run variable cost and t is the percentage price increase of interest.[4]

The concept of critical elasticity is operational because merger policy relates to firms with substantial sunk costs, and because firms with substantial sunk costs charge prices that tend to be well in excess of short-run variable costs. Without sunk costs, antitrust concerns are mitigated by ease of entry. Without prices above short-run variable costs, firms with sunk costs would not exist.

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[3] United States Department of Justice and Federal Trade Commission, *Horizontal Merger Guidelines* (Revised April 1997). Critical elasticity analysis played an important role in the recent case FTC v. Whole Foods Market, Inc., 502 F. Supp. 2d 1 (D.D.C. 2007). In the *Whole Foods* litigation, the reports filed by Defendant's expert (David Scheffman) and Plaintiff's expert (Kevin Murphy) reprise some of the same issues discussed in this paper. For an overview of that case, *see* Deborah Feinstein & Michael Bernstein, *A Perspective on the Whole Foods Decision: Would the Most Important Evidence Please Stand Up?*, COMPETITION POL'Y INT'L, (Spring 2008). *See also* the discussion of critical elasticity in Ken Heyer & Nicholas Hill, *The Year in Review: Economics at the Antitrust Division, 2007-2008*, REV. INDUS. ORG., (November 2008).

[4] Our derivation of this "would" elasticity differs from previous treatments in that it is based on profit maximization. The appropriate way to compute the critical elasticity follows from what the monopolist would do, given profit maximization, rather than from what the monopolist could do, given that profits are no less after the price increase than before. *See* Michael Baumann & Paul Godek, *Could and Would Understood: Critical Elasticities and the Merger Guidelines*, ANTITRUST BULL., (Winter 1995). Farrell & Shapiro (2008) discuss the same issue, but they seem to have been unaware of our derivation.

III. AN ALTERNATIVE VIEW

An alternative view proposes to use a calculated critical elasticity to infer information about the actual demand elasticity:

> When gross margins are large, defense claims that the elasticity of demand is high should be treated with a healthy dose of skepticism. More specifically, we advocate an approach under which there is a presumption that high gross margins go along with a low elasticity of demand faced by the hypothetical monopolist.[5]

and

> Here, one can make inferences about demand sensitivity, as gauged by a real firm based on its premerger choice of price. In particular, if (before the merger) a firm chooses a high margin on its product, the firm evidently thinks that demand for its product is not very sensitive to price.[6]

The gross margin is the difference between price and marginal cost, relative to the price, which in economic theory is equal to the inverse of the elasticity of demand facing the firm and is known as the Lerner index. In other words, this approach proposes to infer information about an actual elasticity from the critical elasticity calculation.

Those statements and the ensuing analyses are plausible, however, only if the critical elasticity calculation is based on the true marginal cost of the firm, as economic theory defines marginal cost. But economic marginal cost is not easily ascertained. If it were easy to know marginal cost, either short-run or long-run, then determining the elasticity of demand would be easy, and it isn't.

The inappropriateness of inferring market power from accounting costs, we thought, had been long ago established and generally accepted, primarily due to the observations of Frank Fisher. Professor Fisher analyzed and condemned the use of accounting data to infer the Lerner Index:

> The profits-sales ratio is an unreliable estimate of the Lerner Index. Simulated examples show that the errors involved in using it may be large in practice, … even the direction of error cannot be easily determined, nor is there a simple way to recast profits/sales so as to recover the Lerner index from accounting data.[7]

Timothy Bresnahan's observations about this issue are also directly relevant. Professor Bresnahan described one of the central ideas of what he labels the "New Empirical Industrial Organization" ("NEIO") as follows:

> Firms' price-cost margins are not taken to be observables; economic marginal cost ("MC") cannot be directly or straightforwardly observed. The analyst infers MC from firm behavior, uses differences between closely related markets to trace

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[5] Katz & Shapiro, *Critical Loss*, *supra* note 2.

[6] Farrell & Shapiro, *Improving Critical Loss*, *supra* note 2. These two papers also include the concept of a "diversion ratio" in the analysis. That concept is an extension that requires the resolution of the critical loss issue and is not necessary to the discussion here.

[7] Franklin Fisher, *On the Misuse of the Profits-Sales Ratio to Infer Monopoly Power*, RAND, (Fall 1987); reprinted in INDUSTRIAL ORGANIZATION, ECONOMICS, AND THE LAW, Ch. 6, (1991).

> the effects of changes in MC, or comes to a quantification of market power without measuring cost at all.[8]

Professor Bresnahan further noted that the NEIO is motivated by "dissatisfactions" over maintained hypotheses in the structure conduct performance paradigm, one of which is that "economic price-cost margins (performance) could be directly observed in the accounting data."[9]

We observe that the critical elasticity calculation is based on something better understood and more readily observed in the business world than marginal cost. In practice, actual critical elasticity analysis is based on the margin over short-run variable cost.[10] We will use the term variable-cost margin (or "VCM") to refer to *the excess of price over short-run average variable cost*. The margin over short-run variable cost, also known as quasi-rent, determines whether a firm is earning enough to justify its investment in fixed assets. A brief description of quasi-rents is worth recalling here:

> A quasi-rent is the return to a durable and specialized productive instrument. … In the long run—in a period long enough to build new instruments or wear out old ones—the return to the instrument must equal the current rate of return on capital (with appropriate allowance for risk). If the machine's quasi-rents are less than interest plus depreciation, it will not be replaced; if the quasi-rents exceed interest plus depreciation, more will be built until equilibrium is restored. The long-run net return on capital goods must yield the appropriate interest rate; their short-run gross return is a quasi-rent.[11]

Whether a firm's VCM is sufficient to justify the investment made to acquire and maintain capital goods helps to determine whether such investments were worthwhile and whether similar investments will be made in the future. And capital goods are more appropriately considered to include all sunk costs, such as advertising, research and development, and the failures necessary to achieve success (known as "dry holes" in not only the oil industry).

Critical elasticity analysis, based as it is on variable cost, does not allow for the inference of market power or the calculation of a Lerner index. In other words, the existence of sunk costs does not imply the existence of market power. And a calculated critical elasticity does not reveal information about an actual elasticity.

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[8] Timothy F. Bresnahan, *Empirical Methods of Industries with Market Power*, in HANDBOOK OF INDUSTRIAL ORGANIZATION, 1012, (Schmalensee & Willig, eds.), (1989).

[9] *Id*., pp. 1012-1013. Some of the discussion here is reiterated in JEFFREY M. PERLOFF, LARRY S. KARP, & AMOS GOLAN, ESTIMATING MARKET POWER AND STRATEGIES, Ch. 2 (2007).

[10] We do not suggest that the calculation of the appropriate variable cost margin is trivial. The determination of which costs to include can be problematic. For a related discussion, *see* William Baumol, *Predation and the Logic of the Average Variable Cost Test*, J.L. & ECON., (April 1996).

[11] GEORGE STIGLER, THE THEORY OF PRICE, 4th Edition, 263, (1987).

IV. CALCULATING THE NEW CRITICAL ELASTICITY

The alternative view, however, has revealed a flaw in the previous analysis of the subject. The critical elasticity formulae in the literature depend on the assumption that marginal costs and average variable costs are constant—and therefore equal. Under that assumption, knowing average variable cost would be equivalent to knowing marginal cost.[12]

A more realistic paradigm would acknowledge that marginal cost is unknown—except that it is not equal to average variable cost and is, therefore, upward sloping at the firm's profit-maximizing level of output: in other words, the standard textbook description of cost curves.[13] When thinking about incremental costs, it should be remembered that production does not sell itself. Short-run marginal cost is the cost of producing *and selling* incremental output. It seems likely that even if production costs are fairly stable, selling costs—the costs of obtaining another sale—begin to increase at some point. That paradigm would seem to be a fair characterization of how competing firms with relatively constant costs of production arrive at an equilibrium distribution of sales.

It is possible to derive a revised critical elasticity formula based on that economic paradigm of rising marginal cost. Assuming linearity of demand and marginal cost the critical elasticity formula becomes:

$$\frac{-1+\sqrt{1+(2m/t)}}{2m}$$

where, as before, m is the initial margin over average variable costs, the VCM, and t is the percentage price increase of interest. (See the Appendix for the derivation.) The derivation of this formula assumes that price is equal to marginal cost at the initial equilibrium. That assumption is employed not to imply the equality between marginal cost and price, but rather because that assumption produces an upper bound on the critical elasticity. That is, if the initial price is actually above marginal cost, the formula overstates the critical elasticity. (Proof is available from the authors.)

Note that the new formula derives uniformly and substantially higher critical elasticities than those derived from previous models. The following table shows various critical elasticities for price increase thresholds of 5 percent, 10 percent, and 15 percent, for both the increasing-marginal-cost model derived here and the conventional constant-cost model. (Again, both formulae are "would" elasticities; they are based on the assumption of profit maximization.) For the values shown in the table, the new critical elasticity is 1.1 to 2.8 times greater than the

[12] Some of the defenders of the original approach to critical elasticity, in desperation it would seem, have invoked the theory of the "kinked demand curve"—a theory that was debunked more than 60 years ago. *See* George Stigler, *The Kinky Oligopoly Demand Curve and Rigid Prices*, J. POL. ECON., (October 1947). But wait, there's more. Thirty years later George Stigler again singled out this theory for its unjustified longevity in economics textbooks. *See* George Stigler, *The Literature of Economics: The Case of the Kinked Oligopoly Demand Curve*, ECON. INQUIRY, (April 1978). It seems that bad theories never die—ever.

[13] *See*, for example, JACK HIRSHLEIFER, AMIHAI GLAZER, & DAVID HIRSHLEIFER, PRICE THEORY AND APPLICATIONS: DECISIONS, MARKETS, AND INFORMATION, 7th Edition, Ch. 7 (2005) or any other price theory textbook.

conventional approach would indicate. This result should not be surprising. The previous critical elasticity model has the hypothetical monopolist losing the same profit "margin" on all sales, while the new model has the hypothetical monopolist losing very little profit margin on its initial lost sales.

Critical Elasticities

Assumption	*Price Increase*	*Variable Cost Margin*								
		10%	*20%*	*30%*	*40%*	*50%*	*60%*	*70%*	*80%*	*90%*
	5%	5.0	3.3	2.5	2.0	1.7	1.4	1.3	1.1	1.0
Constant Marginal Cost	*10%*	3.3	2.5	2.0	1.7	1.4	1.3	1.1	1.0	0.9
	15%	2.5	2.0	1.7	1.4	1.3	1.1	1.0	0.9	0.8
	5%	6.2	5.0	4.3	3.9	3.6	3.3	3.1	3.0	2.8
Increasing Marginal Cost	*10%*	3.7	3.1	2.7	2.5	2.3	2.2	2.1	2.0	1.9
	15%	2.6	2.3	2.1	1.9	1.8	1.7	1.6	1.5	1.4

It follows that, since higher critical elasticities imply that more sales must be lost to deter a given price increase, this new approach will tend to result in narrower market definitions. In sum, the recent critique of critical elasticity, while misguided, has revealed a need for a new formulation of critical elasticity.

V. CONCLUSION

Based on our analysis, parties seeking to assist the antitrust agencies or the courts in defining relevant markets should continue to feel comfortable making the usual critical elasticity argument, which is that higher variable cost margins indicate that fewer lost sales are needed to deter a price increase. (And it is easier to lose fewer sales than more sales.) It should be understood, however, that the conventional critical elasticity formula is not appropriate. It should also be understood that calculated critical elasticities are not informative about actual elasticities.

VI. APPENDIX: A NEW CRITICAL ELASTICITY

Determining whether a hypothetical monopolist would raise its price a certain threshold percentage implies a comparison of the profit-maximizing monopoly price, P_M, and the initial price, P_0. In percentage terms, the price increase is less than the price increase threshold, t, if $(P_M – P_0) / P_0 < t$. The critical elasticity is the elasticity of demand at P_0 that is just high enough to prevent a monopolist from increasing price by t. Here, the critical elasticity is derived under the assumption that the marginal cost and demand functions are linear.

A. Marginal Cost

Assuming linearity, marginal cost (*MC)* can be written

$$MC = \alpha + \beta Q, \tag{1}$$

where α and β are positive constants and Q is quantity. This assumption implies that total cost (TC) is a quadratic function:

$$TC = \gamma + \alpha Q + \frac{1}{2}\beta Q^2 . \tag{2}$$

And variable cost (VC) equals:

$$VC = \alpha Q + \frac{1}{2}\beta Q^2 . \tag{3}$$

Average variable cost (AVC) is equal to VC divided by Q:

$$AVC = \frac{VC}{Q} = \alpha + \frac{1}{2}\beta Q . \tag{4}$$

Taking the difference between marginal cost and average variable cost derives:

$$\beta = 2\left(\frac{MC - AVC}{Q}\right). \tag{5}$$

Initial quantity, marginal cost, and average variable cost are denoted as Q_0, MC_0, and AVC_0, respectively; equations (1) and (5) imply that marginal cost at any quantity is given by the following:

$$MC(Q) = MC_0 + 2\left(\frac{MC_0 - AVC_0}{Q_0}\right)(Q - Q_0). \qquad (6)$$

B. Demand

Assuming linear demand, the inverse demand function can be written

$$P = A - BQ \qquad (7)$$

where P is price, Q is quantity, and A and B are positive constants. Let ε denote the elasticity of demand at the initial equilibrium stated as a positive value:

$$\varepsilon = -\frac{\partial Q}{\partial P}\frac{P_0}{Q_0} = -\frac{P_0}{-BQ_0} = \frac{P_0}{BQ_0}, \qquad (8)$$

where Q_0 and P_0 are the initial quantity and price. Together, (7) and (8) imply that

$$B = \frac{P_0}{\varepsilon Q_0} \text{ and}$$

$$A = P_0 + BQ_0 = P_0\left(1 + \frac{1}{\varepsilon}\right). \qquad (9)$$

C. Profit maximization

Total revenue is price times quantity and price is given by equation (7), so marginal revenue can be written as

$$MR = \frac{\partial(PQ)}{\partial Q} = \frac{\partial((A - BQ)Q)}{\partial Q} = A - 2BQ.$$

Substituting for A and B using (9) yields

$$MR = P_0\left(1 + \frac{1}{\varepsilon}\right) - 2\frac{P_0}{\varepsilon Q_0}Q. \qquad (10)$$

Applying equation (6) and assuming that the market is initially competitive, $MC_0 = P_0$, yields the following expression for marginal cost:

$$MC = P_0 + 2\left(\frac{P_0 - AVC_0}{Q_0}\right)(Q - Q_0)$$

or

$$MC = P_0 + 2\frac{P_0}{Q_0}mQ - 2P_0 m \tag{11}$$

where

$$m = \left(\frac{P_0 - AVC_0}{P_0}\right)$$

is the average variable cost margin (*VCM*) at the initial equilibrium.

Setting $MR = MC$ and applying equations (10) and (11) yields the following first-order condition for profit maximization:

$$P_0\left(1+\frac{1}{\varepsilon}\right) - 2\frac{P_0}{\varepsilon Q_0}Q = P_0 + 2\frac{P_0}{Q_0}mQ - 2P_0 m\,.$$

Grouping terms, dividing through by P_0, and multiplying through by ε gives

$$\frac{2}{Q_0}(1+m\varepsilon)Q = 1 + 2m\varepsilon\,,$$

which implies that the profit-maximizing quantity, Q_M, is given by

$$Q_M = \left(\frac{Q_0}{2}\right)\left(\frac{1+2m\varepsilon}{1+m\varepsilon}\right). \tag{12}$$

The profit-maximizing price, P_M, is given by $P_M = A - BQ_M$. By substitution using equations (9) and (12):

$$P_M = P_0\left(1+\frac{1}{\varepsilon}\right) - \frac{P_0}{\varepsilon Q_0}\left(\left(\frac{Q_0}{2}\right)\left(\frac{1+2m\varepsilon}{1+m\varepsilon}\right)\right) = P_0\left(1+\frac{1}{\varepsilon} - \frac{1+2m\varepsilon}{2\varepsilon(1+m\varepsilon)}\right)$$

or

$$P_M = P_0\left(1+\frac{1}{2\varepsilon(1+m\varepsilon)}\right). \tag{13}$$

D. Critical elasticity

The critical elasticity is the lowest elasticity of demand at P_0 such that a hypothetical monopolist would not increase price by t. Using (13), the percentage increase in price from the initial price, P_0, to the profit-maximizing price, P_M, is

$$\frac{P_M - P_0}{P_0} = \frac{P_0\left(1+\frac{1}{2\varepsilon(1+m\varepsilon)}\right)-P_0}{P_0} = \frac{1}{2\varepsilon(1+m\varepsilon)}. \qquad (14)$$

For the percentage increase in price to be less than the critical value, t, the following condition must hold:

$$\frac{1}{2\varepsilon(1+m\varepsilon)} < t$$

or

$$m\varepsilon^2 + \varepsilon - \frac{1}{2t} > 0. \qquad (15)$$

Solving the quadratic equation (15) for ε and taking the positive root gives the following value for the critical would elasticity:

$$\varepsilon > \frac{-1+\sqrt{1+\frac{2m}{t}}}{2m}. \qquad (16)$$