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Time-Series Models for Estimating Economic Damages in Antitrust (and Other) Litigation: The Relative Merits of Predictive versus Dummy-Variable Approaches

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I. INTRODUCTION

Economic damages can arise from a variety of bad acts: a breach of contract, a patent violation, an oil-spill, or a price-fixing conspiracy, for examples. The calculation of economic damages involves the description of a "but-for world." The but-for world implies a set of economic values—such as profits or quantities or prices—that would have prevailed but-for the alleged bad acts. These but-for values become the basis for calculating damages.

A but-for world cannot be demonstrated with certainty because it never occurs; it is a hypothetical entity. Expert economists play a crucial role in the disputes over economic damages because of their ability to model and estimate what would have happened in a particular economic environment if one aspect of that environment had been different. What would profits have been if a contract or a patent had not been violated? What would fish harvests have been if not for an oil-spill? What would prices have been absent a price-fixing conspiracy?

This exercise often involves the comparison of relevant economic values across two distinct time periods, the damage period and the benchmark or clean period. And the basic exercise, describing and analyzing a but-for world, often involves time-series data—empirical observations that are generated at regular or irregular intervals over time. For example, in a price-fixing conspiracy, regression analysis is typically employed to estimate the historical relationship between actual transaction prices and various explanatory variables, such as production costs and the level of demand for the product. That econometrically derived relationship is used to generate but-for prices within the damage period.[2]

Here I examine the properties of two alternative approaches to implementing time-series damages models. The two approaches will be referred to as "predictive" and "dummy-variable."[3] The comparison between the two approaches also serves as a reminder of the potential problem

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[2] Comparing the but-for prices to the actual prices, to determine whether there are statistically and economically significant differences, is the final step of the process. Descriptions of regression analysis and its role in litigation can be found in Daniel L. Rubinfeld, *Reference Guide on Multiple Regression*, REFERENCE MANUAL ON SCIENTIFIC EVIDENCE, 2nd Ed. (2000) and American Bar Association, Section of Antitrust Law, ECONOMETRICS: LEGAL, PRACTICAL, AND TECHNICAL ISSUES (2005).

[3] A recent comparison of these approaches and a strong advocation of the predictive approach is Halbert White, Robert Marshall, & Pauline Kennedy, *The Measurement of Economic Damages in Antitrust Civil Litigation*, ANTITRUST SECTION ECON. COMM. NEWSLETTER (Spring 2006) (hereinafter "White"). That paper suggests the term "predictive," which I adopt even though both approaches predict but-for prices. The methodologies underlying these approaches are described in standard econometric textbooks.

to which time-series models are susceptible: the crucial but sometimes forgotten "spurious regression" problem.

II. THE TWO METHODS

The dummy-variable approach uses all of the available data, from both the damage period and the benchmark period, to estimate the regression model. Included in the model is a binary (0,1) variable, commonly referred to as a dummy variable, to indicate whether a given data point occurs in the damage period or the benchmark period. That is, the dummy variable is 1 for observations in the damage period and 0 for observations in the benchmark period. The regression coefficient for the dummy variable indicates the extent to which the dependent variable is higher or lower—on average and controlling for all of the factors included in the regression model—in the damage period relative to the benchmark period. In this model, the but-for values are the predicted values from the regression model minus the dummy-variable coefficient.

The predictive approach, in contrast, uses data only from the benchmark period to estimate the regression model. That model is then used to predict but-for values for the dependent variable during the damage period. Values for the control variables during the damage period are "plugged into" the model to generate predicted but-for values. The differences between the predicted and actual values are the calculated effects of being in the damage period, which are then summed to establish an aggregate indicator of damages.

III. CRITIQUES

Proponents of the predictive approach argue for its use based on their view of a perceived flaw in the dummy-variable approach. The supposed flaw is described in the context of determining but-for prices in a price-fixing case:

> Cost and demand shifters generally have different predictive coefficients in cartel and non-cartel regimes, due to the varying responsiveness of the firms in the industry to these factors between regimes. …

> For example, firms engaging in coordinated behavior may be engaging in supply restrictions to keep prices inflated and so be less responsive to increases in customer demand in terms of the quantity brought to market.[4]

The point is a fair one, though its relevance is overstated. The dummy-variable approach implicitly assumes that the relationships between the control variables and the dependent variable will be approximately the same in both the damage and the benchmark periods, or at least that any differences in the relationships will not introduce a bias in the estimate of the dummy-variable coefficient.

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[4] *Id.* at 19, 22. White is at points misleading in that it offers general criticisms that would apply to any econometric approach and then claims that the criticisms do not apply to the predictive approach. For example, the paper notes that "regardless of whether or not a dummy variable approach is used, misleading results will arise from including variables impacted by the cartel" and that "misleading results can arise from including irrelevant variables whose time paths have shapes similar to that of the price (dependent variable) path." (Page 22.) These statements are correct and apply equally well to either approach. Nonetheless, the paper claims that the predictive approach "excludes variables impacted by the cartel" and "excludes irrelevant variables." (Page 22.) Finally, the claim that the dummy-variable approach "is almost always misleading and highly vulnerable to manipulation" (page 18) has no basis.

As noted, however, the point is overstated. Coefficients will never be precisely the same in two subsets of data, but there is no reason to expect that the differences will be significant or that even significant differences will introduce a bias into the estimate of the dummy-variable coefficient. Furthermore, there is a simple test for whether or not the supposed problem (of different coefficients between the two periods) is statistically significant. A standard statistical test, known as the Chow test, can be used to determine whether the two sub-periods of data reflect the same basic underlying relationships.[5]

Even if a Chow test identifies a statistically significant difference, that is still not necessarily reason enough to abandon the dummy-variable approach. First, statistical significance does not necessarily imply economic significance.[6] That is, slight differences in coefficients can be statistically significant without having any material impact on the estimate of the dummy-variable coefficient. Second, the difference in the two models may still leave the dummy-variable coefficient unbiased as an estimator of damages. Indeed, there would seem to be no *a priori* reason to expect otherwise. Finally, the results of the Chow test may indicate changes in the estimated relationships that are unrelated to the issues in the litigation, but that reflect other factors not controlled for in the model—a circumstance that would be problematic for either approach.

Similarly, proponents of the dummy-variable approach argue for its use based on their views of a perceived flaw in the predictive approach. Because the predictive approach uses only benchmark-period data to estimate the econometric model, it is intentionally ignoring relevant and potentially valuable information. This critique will be particularly relevant depending on the amounts of data available in each period. Let's say, for example, that data in the benchmark period provides only a small sample. Using a small sample introduces actual imprecision in order to avoid a possible problem. Alternatively, let's say that the benchmark period is small relative to the damage period. To rely, for example, on one year's worth of benchmark data to generate five years' worth of predicted prices assumes a perhaps unreasonable amount of robustness and stability in the estimated benchmark model.

IV. SPURIOSITY

At this level, the debate may seem peculiar. After all, shouldn't the two methods generate similar results? As a general matter, the answer is no. If there exists: a) an agreed upon set of independent variables with stable relationships in the regression model, b) a damage period that is bounded by benchmark periods both before and after, and c) a brief damage period relative to the benchmark periods, then the two approaches would likely generate similar results. In the real world, however, those conditions rarely apply.

Is there any reason to expect that one method or the other is more prone to bias or manipulation? Any econometric analysis is susceptible to manipulation by the inclusion of irrelevant variables. But this is especially problematic in time-series models because of the well-known spurious-regression problem: the tendency to find significant correlation or regression

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[5] The Chow test is described in standard econometrics textbooks. Since the dummy-variable approach assumes that the estimated relationships between the two periods will be similar, except for a shift in the intercept term, the Chow test is operative. *See* WILLIAM H. GREENE, ECONOMETRIC ANALYSIS, 5th Ed. Ch. 7-8 (2003) and Jan Kmenta, *Elements of Econometrics*, 2nd Ed. Ch. 10 (1986).

[6] For an insightful discussion of the tendency to confuse statistical and economic significance, *see* STEPHEN T. ZILIAK & DEIRDRE N. MCCLOSKEY, THE CULT OF STATISTICAL SIGNIFICANCE (2008).

results when relating "non-stationary" time series. The term non-stationary as used here refers to time series that have different means in different sub-periods—in other words, series that are trending up or down over time.[7] The tendency to find spurious correlation in time series data has been recognized since Yule's seminal paper in 1926. In that article Yule noted:

> It is fairly familiar knowledge that we sometimes obtain between quantities varying with the time (time-variables) quite high correlations to which we cannot attach any physical significance whatever, although under the ordinary test the correlation would be held to be certainly "significant."[8]

In 1974, Granger & Newbold reminded the economics profession that this problem was being neglected and was generating nonsense results in regression analysis.[9] More recently, Phillips summed up the issue well:

> The simple heuristic explanation for phenomena of this type is that conventional statistical tests do nothing more than reveal the presence of a trend in the dependent variable by making the fitted coefficients significant for all regressors that themselves have trends. Thus, the commonality for trending mechanisms in data is the source of these spurious regressions.[10]

A stylized demonstration of the problem is presented here. The following chart shows a dependent variable; call it the price of widgets. The explanatory variable is some economic measure plausibly, but not actually, related to the price of widgets; call it the interest rate. The data are constructed so that the price of widgets has virtually the same average value in the benchmark period and the damage period. A dummy-variable model correctly finds no significant relationship between interest rates and widgets and assigns a value not significantly different from zero for the dummy-variable coefficient.[11]

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[7] For a technical discussion of "stationarity," *see* JAMES D. HAMILTON, TIME SERIES ANALYSIS, Ch. 3 (1994).

[8] G. Udny Yule, *Why Do We Sometimes Get Nonsense-Correlations between Time-Series? A Study in Sampling and the Nature of Time-Series*, J. ROYAL STATISTICAL SOC'Y 2 (January 1926). John Aldrich, *Correlations Genuine and Spurious in Pearson and Yule*, 10(4) STATISTICAL SCI. (1995), offers a thorough discussion of the history of the subject. The anonymous "Student" (William Gosset) seems to have been the first to use the term "spurious" in a statistical context. *See* Student, *The Elimination of Spurious Correlation Due to Position in Time or Space*, 10(1) BIOMETRIKA (1914).

[9] C.W.J. Granger & Paul Newbold, *Spurious Regressions in Econometrics*, 2 J. ECONOMETRICS (1974). *See also* C.W.J. GRANGER & PAUL NEWBOLD, FORECASTING ECONOMIC TIME SERIES Ch. 6 (1977).

[10] Peter C.B. Phillips, *New Tools for Understanding Spurious Regressions*, ECONOMETRICA 1300 (November 1998). Phillips points out that adjusting standard errors for autocorrelation does not eliminate the problem because spurious relationships tend to remain statistically significant. These issues are also reviewed in Peter C.B. Phillips, *Laws and Limits of Econometrics*, ECON. J. (March 2003) and David F. Hendry, *Economic Modeling with Cointegrated Variables: An Overview*, 48(3) OXFORD BULL. ECON. & STATISTICS (1986).
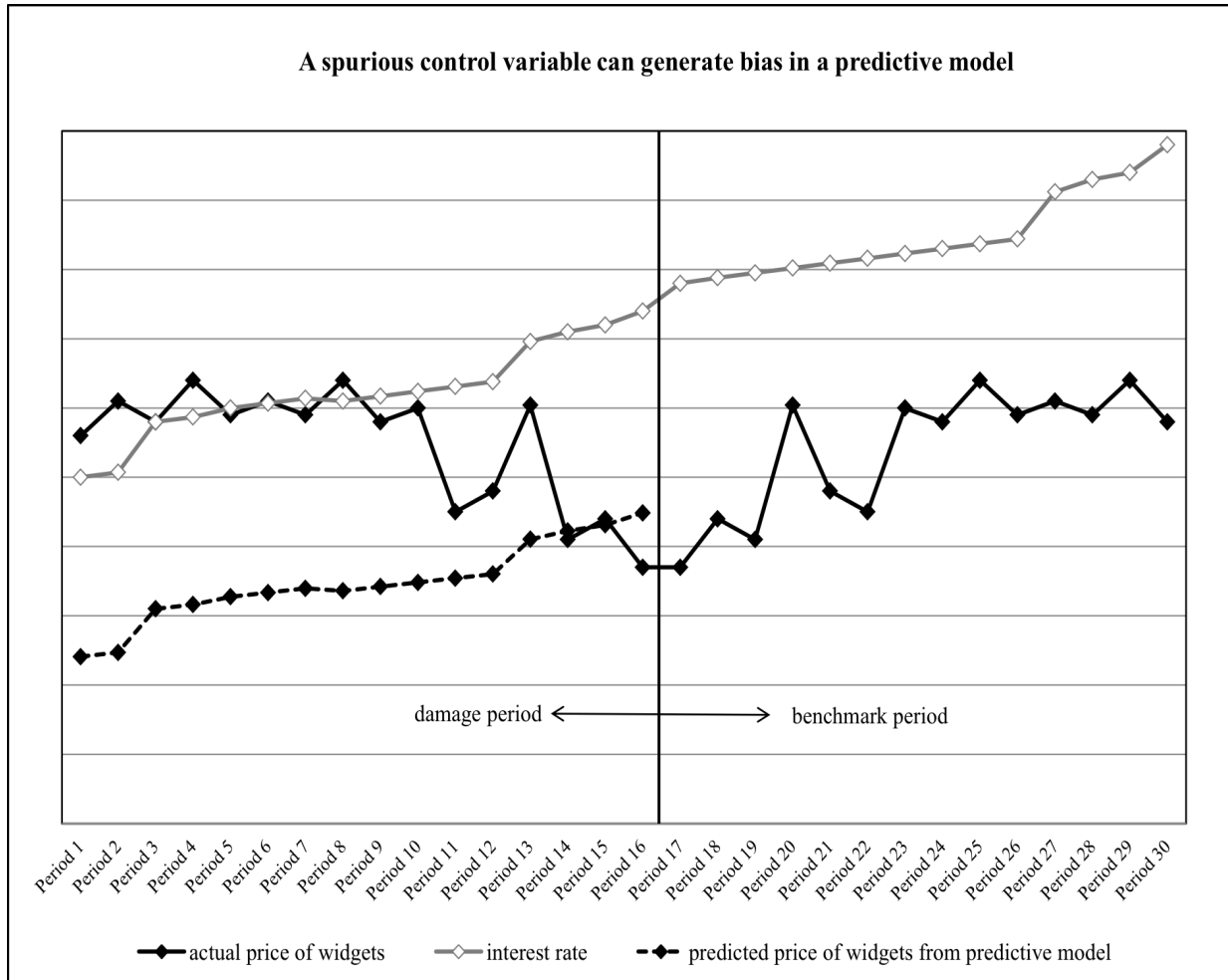
[11] The dummy variable model, which uses all of the available data, is:

widget prices = intercept + interest rate + dummy variable + error term.

The coefficients on the dummy variable and interest rate are not statistically significant. The predictive model, which uses only the data in the benchmark period, is:

widget prices = intercept + interest rate + error term.

In this model, which is used to generate the predicted prices in the chart, the interest rate is spuriously significant.

A spurious control variable can generate bias in a predictive model

damage period ←————————→ benchmark period

——◆—— actual price of widgets ——◇—— interest rate --◆-- predicted price of widgets from predictive model

Period 1, Period 2, Period 3, Period 4, Period 5, Period 6, Period 7, Period 8, Period 9, Period 10, Period 11, Period 12, Period 13, Period 14, Period 15, Period 16, Period 17, Period 18, Period 19, Period 20, Period 21, Period 22, Period 23, Period 24, Period 25, Period 26, Period 27, Period 28, Period 29, Period 30

A predictive model, however, is susceptible to the spurious regression problem in this example because of the coincidence that both interest rates and widget prices have trends in the benchmark period. The dotted black line shows the predicted widget prices using the predictive model, which are significantly below the actual prices. Thus, the predictive model generates a spurious difference between actual and predicted prices, while a dummy-variable model correctly finds no significant difference.

This simple demonstration refutes both the claim that predictive models are not susceptible to manipulation as well as the claim that dummy-variable models are inherently so.[12] The tendency for misspecification, using either approach, is likely to be particularly problematic when there is only one benchmark period, either before or after the damage period, but not both before and after. In any case, the spurious-regression problem remains a formidable concern to practitioners of applied econometrics.

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[12] *See* footnotes 2-3 *supra*. While this stylized example resembles an antitrust price-fixing case, the same issues can arise in any context in which one is comparing economic values across two periods.

V. SUMMARY

In stable well-specified models with ample benchmark data both before and after the damage period—a desirable though atypical situation—the two approaches discussed here should generally produce similar results. But regardless of the circumstances, in the absence of a demonstration that using all of the available the data would substantially bias the estimate of the dummy-variable coefficient, the dummy-variable approach should be the default. The reason is that one should be hesitant to throw away data in any econometric exercise. As data are lost, precision decreases and the tendency for misspecification increases. In sum, those who would ignore relevant data should have to justify the reason for doing so.